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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

No. 212 was also solved by Jeannette Brooks. Two solutions of No. 213 were received from G. B. Hudson.

215. Proposed by EDWIN L. RICH, Lehigh University.

$$\begin{aligned}\text{Solve (1).....} & x/a + y/b + z/c = 3, \\ (2)..... & x/a + b/y + z/c = 3, \\ (3)..... & a/x + y/b + z/c = 3.\end{aligned}$$

I. Solution by J. SCHEFFER, Hagerstown, Md.

Subtracting (3) from (1) and (3) from (2) the results may easily be put in the form

$$\begin{aligned}(z/c)^2 - (x/a - a/x)(z/c) &= 1 \text{.....(4),} \\ (y/b)^2 - (x/a - a/x)(y/b) &= 1 \text{.....(5),}\end{aligned}$$

whence $z/c = x/a$, $-a/x$; $y/b = x/a$, $-a/x$.

By substituting the first pair of values in (1),

$$x = a, y = b, z = c, \text{ and } x = \frac{1}{2}a, y = \frac{1}{2}b, z = \frac{1}{2}c.$$

By substituting the second pair of values we obtain.

$$\begin{aligned}x &= 3a, y = -b/3, z = 3c; \\ x &= -a/3, y = 3b, z = 3c, \\ x &= 3a, y = 3b, z = -c/3.\end{aligned}$$

II. Solution by L. S. SHIVELY, Mt. Morris, Ill., and ELMER SCHUYLER, Brooklyn High School, New York.

Let $x/a = A$, $y/b = B$, and $z/c = C$. Then the original equations become

$$\begin{aligned}A + B + 1/C &= 3 \text{.....(1),} \\ A + 1/B + C &= 3 \text{.....(2),} \\ 1/A + B + C &= 3 \text{.....(3).}\end{aligned}$$

Subtracting (2) from (1) and $B - 1/B = C - 1/C$, hence $B = C$, $-1/C$.

It can similarly be shown that

$$\begin{aligned}A &= B = C \text{.....(4), and that} \\ A &= -1/B; B = -1/C; C = -1/A \text{.....(5).}\end{aligned}$$

From (4) and (1), $2A + 1/A = 3$. The roots of this quadratic are $\frac{1}{2}$ and 1.

$\therefore A = B = C = \frac{1}{2}$, and $x = \frac{1}{2}a$, $y = \frac{1}{2}b$, $z = \frac{1}{2}c$; $A = B = C = 1$, and $x = a$, $y = b$, $z = c$.

From (5) and (1) it is seen that $A=3$, $B=-\frac{1}{3}$, $C=3$.

$\therefore x=3a$, $y=-\frac{1}{3}b$, $z=3C$.

In like manner, $x=3a$, $y=3b$, $z=-\frac{1}{3}c$, and $x=-\frac{1}{3}a$, $y=3b$, $z=3c$.

Also solved by M. E. Graber, Grace M. Bareis, E. L. Sherwood, Christian Hornung, F. P. Matz, G. B. M. Zerr, and the Proposer.

216. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

Express by radicals the roots of $x^6 + ax^4 + bx^3 + \frac{1}{4}a^2x^2 + \frac{1}{2}abx + c = 0$.

I. Solution by E. L. SHERWOOD.

$x^6 + ax^4 + \frac{1}{4}a^2x^2 + bx^3 + \frac{1}{2}abx + c = 0$, $(x^3 + \frac{1}{2}ax)^2 + b(x^3 + \frac{1}{2}ax) + c = 0$, whence we have, by solving the quadratic

$$x^3 + \frac{1}{2}ax + \frac{2b - b^2 + 4c}{4} = 0, \text{ or } x^3 + \frac{1}{2}ax + \frac{2b + b^2 - 4c}{4} = 0,$$

whence by Cardan's method, Burnside and Panton, p. 108,

$$x = \sqrt[3]{p} + \frac{-H}{\sqrt[3]{p}}, \quad \omega \sqrt[3]{p} - \frac{H}{\omega \sqrt[3]{p}}, \quad \omega^2 \sqrt[3]{p} - \frac{H}{\omega^2 \sqrt[3]{p}}$$

where $p = \frac{1}{2}[\sqrt{(G^2 + 4H^3)} - G]$, and $G = \frac{2b - b^2 + 4c}{4}$ or $\frac{2b + b^2 - 4c}{4}$, $H = \frac{1}{6}a$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Write the equation as follows:

$$x^2(x^2 + \frac{1}{2}a)^2 + bx(x^2 + \frac{1}{2}a) + c = 0.$$

Let $x(x^2 + \frac{1}{2}a) = x^3 + \frac{1}{2}ax = y$. $\therefore y^2 + by + c = 0$.

$$\therefore y = \frac{-b \pm \sqrt{(b^2 - 4c)}}{2}.$$

Let ω be an imaginary cube root of unity, and let m, n be the roots of $t^2 - yt - a^3/216 = 0$.

$$\therefore x = m + n, \quad x = \omega m + \omega^2 n, \quad x = \omega^2 m + \omega n.$$

As y has two values, the six values of x are expressed as radicals.

Also solved by J. Scheffer, G. W. Greenwood, Elmer Schuyler, F. P. Matz, and the Proposer.

217. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

Find the condition that $E \equiv x^5 - bx^3 + cx^2 + dx - e$ shall be the product of a complete square and a complete cube.

I. Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

The factors must be of the form

$$(x^2 - 2ax + a^2)(x^3 + 2ax^2 + \frac{4a^2x}{3} + \frac{8a^3}{27}),$$